TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TM No. 338

RECENT PROGRESS IN THE THEORETICAL DEDUCTION OF AIRPLANE WINGS

By M. Panetti

From "Revista Aeronautica," July, 1925

Washington
November, 1925

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1. The designing of an airplane, even in its general lines, requires an accurate knowledge of the aerodynamic properties of its wings, as expressed in the polar diagram.

It often happens that the experimental wing profiles whose characteristics are known although numerous, do not satisfy the requirements. Thus the biconvex profiles, particularly adapted to very swift flight by their very small resistance, have not yet been tested in a sufficient variety of shapes, as regards the curvature of the two sides, to serve as safe guides in designing wings for the various speeds of climbing and horizontal flight.

On the other hand, the increasingly numerous uses of the monoplane, in which the thickness of the wing is directly related to the static functioning and therefore to the strength of the structure, have caused to be included in the problems of capital importance the wing with a profile variable from one tip to the other. We have at our disposal a decidedly insufficient number of experimental researches for wings of this kind, especially since the law of variation of the thick-

* From "Revista Aeronautica," July, 1925, pp. 69-75.

ness and of the profile, according to the plan of the supporting system, can vary infinitely for the different types and would therefore require special experiments for each individual case.

2. It is therefore important to continue the theoretical researches on the aerodynamic phenomena of wings, in order to determine by calculation, at least approximately, their fundamental characteristics and to increase and formulate our knowledge in this field, so as to facilitate the necessary calculations.

3. Beginning with the first problem, that of wings with a uniform cross-section or profile, it is known that we are today in possession of quite simple methods for deducing theoretically the diagram of the lift and of the displacements of the center of pressure on Joukowski profiles, deducible from the circle with a conform transformation of the type

$$z = \zeta + \frac{h^2}{\zeta} .$$

These profiles are easily recognized by the almost symmetrical shape of the leading edge and because, being derived from highly cambered types, the lower side is considerably curved and is concave near the trailing edge, where it is consequently very thin. Any one of these profiles is defined by two parameters. The first parameter determines the camber of

the mean line of the wing and consequently the dyssymmetry between the upper and lower side. The second parameter determines the thickness. The zero value of the first parameter corresponds to the symmetrical profile and its values can be of interest up to 0.25. The zero value of the second parameter corresponds to zero thickness, so that the wing section is reduced to a simple arc of a circle and its values can be of interest up to 0.2.

4. It is important for a designer, who wishes to choose the best wing for a given airplane, to know how to analyze a given profile, resembling this type, in order to be able to plot the polar curve. Let us recall therefore how the analysis of a Joukowski profile is made on the diagram of the axial line, coinciding with the arc of a circle, to which the profile is reduced when the thickness parameter becomes zero. This arc connects the trailing edge of the profile with the center of curvature of the leading edge, at the point of maximum curvature, and passes approximately through the central point of the thickness of the wing, measured at equal distances from the ends of the axial line. This mean thickness of the profile renders it possible to calculate the thickness parameter ϵ, by means of the ratio R of said thickness to the chord subtending each half of the axial line. This purpose is served by the formula

$$R = \frac{1 + \epsilon}{1 + 2\epsilon} 2\epsilon \ .$$

From the thickness ϵ we find the location of the center and of the focus of the wing and, consequently, the enveloping parabola of its lift coefficients at various angles of attack. From the axial line, connecting the trailing edge with the vertex, we deduce the direction of zero lift, fundamental for determining the law of lift increase. The lift increases, within the practical limits of flight, in proportion to the absolute angle of attack α, which the direction of motion makes with the line of zero lift. The theoretical coefficient of proportionality c' so closely approximates the one derived from the best experiments that, from this viewpoint, there remains little to be desired. Its approximation formula is

$$c' = \frac{2\pi}{1 + \frac{2}{\lambda}},$$

in which λ is the aspect ratio of the wing. By means of this formula, it is easy to find the parabola to which the polar curve of a wing would be reduced, were it not for the detachment of the streamline from the rear portion of the profile, due to the viscosity of the air.

It suffices therefore to retain the constant intrinsic resistance (by fixing it at the well known values c_o, which depend on the thickness of the wing considerably more than on the shape of its profile) and to plot the polar parabola with abscissas and ordinates given respectively by the well known

formulas

$$c_x = c_o + \frac{1}{\pi\lambda} (c_y)^2 \qquad c_y = c'\alpha$$

5. The element of calculation, thus derived, is of value in the resolution of the first group of fundamental problems, such as the determination of the maximum velocity, of the minimum climbing time, of the altitude limit or "ceiling" and of the minimum angle of attack for horizontal flight.

The elements of calculation regarding slow flight and, consequently, the maximum coefficients of lift are, nevertheless, empirical, since, in correspondence with these, the effective polar differs very appreciably from the theoretical, due to the phenomenon of viscous flow. At present, the theory is too complex to enable the determination of the point of detachment of the lines of flow from the top of the wing profile. We know, however, that, for Joukowski profiles and probably for all profiles which have been adopted in practice, this detachment produces greater coefficients of lift for very thick wings. In these, however, the difficultly determined detachment is accentuated by a sudden fall in the value of the coefficient of lift, as if the aerodynamic phenomenon passed through one of its critical points, which gave the thick-winged airplanes particular instability in flight at minimum speed.

6. The enumeration of the results, possible to be obtained by an exclusively theoretical method with Joukowski pro-

files, demonstrates how important it would be to make such methods general, that is, to make, for any profile whatever, the same analysis identifying it with certain parameters and deducing from the latter the polar of the wing with a constant profile. Only a few attempts have thus far been made in this direction.

In addition to the praiseworthy classification of wing profiles undertaken by the engineering section of the Italian Air Service, in an opportune and comprehensive tabulation of the results of the various laboratories, mention should be made of Geckeler's article published three years ago in "Zeitschrift für Flugtechnik und Motorluftschiffahrt," which was based on a double conform representation. After plotting the double set of ellipses and confocal hyperbolas, with which there is conformably represented the system of orthogonal trajectories consisting of concentric circles and the bundle of relative rays, he plotted a wing profile with its posterior vertex in one of the foci and with the leading edge encircling the other focus, in such fashion that its contour differs the least possible from an orthogonal trajectory of the set of hyperbolas. On transforming the profile by points, there was produced a curve, approximating a circle, which the author suggests to be considered as an ellipse of slight eccentricity.

From the ellipse he then passes to a circle with a second conform transformation of the known type. The bond thus established renders it possible to plot, for a given angle of

attack, the diagram of the dynamic pressures on the wing profile and therefore solves all problems relative to the lift of the wing in the Euler hypothesis of motion.

7. The problem of the wing with variable cross-section is very important for reasons already enumerated. Its solution, with respect to the phenomenon of lift, consists in the application of the Kutta-Joukowski theorem, combined with that of Prandtl, which confirms the identity of the lift with the circulation and not merely with the formation of free vortices, as a consequence of the fact that the circulation is variable and as a factor which affects the lift of the wing.

According to this principle, if two adjacent portions of a wing, through a difference in the profile and in orientation, develop different lifts, there must be liberated, in correspondence with the cross-section which separates them, a vortical filament in the direction of the relative motion of the air. The intensity of this filament is equal to the difference in circulation between the two contiguous portions. The direction is that which results from considering said vortex as the continuation of the one obtained by the difference of the two adjacent vortices.

The free vortices thus formed generate an induced velocity on the leading edge of the wing, which can be calculated at every point by superposing the effects of the single induced vortices.

This velocity diminishes the apparent angle of attack and consequently reduces the lift which would be calculated for the single sections, if each section were to be considered as part of a wing of infinite length and constant cross-section. Induced drag would be created at the same time.

8. The solution of this problem, by whatever method, cannot be very laborious, since it is a problem in which the results influence the causes. If we adopt a method of successive approximations, we can plot the diagram of the lifts on the assumption that the sections are of infinite length, then deduce from it the intensity of the vortical system and, by means of this, calculate the velocity induced on the whole frontal development of the wing. After next modifying the diagram of the lifts, we repeat the operation until the diagram differs but little from the original. As is always the case in methods of this kind, a few repetitions of the calculation generally suffice to obtain a satisfactory result, which then leads directly to the calculation of the drag.

It is obvious, however, that the whole process must be repeated for each angle of attack, which makes its use tedious. Other methods are therefore being investigated at the Turin Laboratory, which will render it possible to find utilizable factors for different angles of attack and concerning which a report will soon be made. The mathematical problem of the lift of the wing, which appeared, in its beginnings, to be simply

of theoretical interest, is daily becoming more of a technical problem.

9. The important scientific data now at our disposal enable us to consider and discuss qualitatively, even without the aid of formulas, this and other practical questions with great advantage for the better understanding of the phenomenon with which we must become familiar. This very attitude constitutes the merit of modern aerotechnics and the proof of the maturity attained by it, which gives it the right to be counted among the most advanced technical sciences.

10. If, for example, we consider a question much debated in the past, which has points of contact with this problem, we wish to know what effect on the length of a wing is produced by the interruption due to a fuselage incapable of supporting action, or capable only to a negligible degree in comparison with the wing, we can easily find, with the aid of the principles enunciated, the reason for the fact, demonstrated by experiment, that this effect is very small.

In truth, because of the symmetry of the wing plane with reference to the fuselage which interrupts it, the free vortices produced by this interruption are of opposite sign and very similar. These must therefore be neutralized, if not in their effective physical existence, at least in the effects produced at a distance.

Recurring to the other conception of the compensating currents in the plane at right angles to the direction of motion, direct from the lower surface, where there is positive pressure, to the upper surface, where there is negative pressure, we must, however, recognize that, while said currents can be quite strong at the wing tips and diminish the efficiency (marginal phenomenon), in correspondence with the fuselage, these are opposed by the presence of the fuselage in the plane of symmetry of the airplane.

The principles thus far enunciated must not be taken too literally. If the progress of the mathematical principles and their technical elaboration are today very desirable in the field of aeronautics, it is none the less true that the experimental control of their deductions is of the greatest importance. The science of the complicated subject of the dynamics of fluids is still in its infancy. Being too involved in rigorous mathematical research, it runs the risk of being erroneously interpreted. Only the correct equilibrium between the two methods of investigation can assure satisfactory progress. Experiments interpreted by scientific reasoning and conducted for stimulating deductions is always the best way to make progress.

Translation by Dwight M. Miner,
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